COVID-19 Exponential Growth

Geoffrey Downing

At the end of the fall 2020 semester, the Covid-19 global pandemic has taken 1.61 million lives, and there have been 72.1 million cases globally since it was first recognized as a health threat by the U.S. Government on January 28, 2020. At that point in time, there were less than 100 documented cases in the world. The coronavirus spread like wildfire, and the mathematical way to understand the spread of such a virus is using an exponential function. It is my goal with this project to grasp and demonstrate an understanding of the spread of Covid-19 as an exponential function, and to discuss the application of this factual information in decreasing the exponential spread of Covid-19.

In mathematics, an exponential function is defined as a function of the form f(x)=abx where b is a positive real number not equal to 1, and the argument x occurs as an exponent. The World Health Organization describes it slightly differently with respect to viruses, but the graph of an exponential function looks the same, as it increases exponentially, the curve of the graph increases more and more rapidly through the progression of time. WHO, for purposes of describing the exponential spread of Covid-19, has assigned a person who is infected with Covid-19 a reproductive number, or R0, which they believe to be 3. This means that for every one person who gets infected with the virus, that one person will likely infect, on average, three other people. Then those three people will likely each infect three other people. The reproductive number is applied over the course of 5 days from the time the individual is infected. Therefore, starting at one case of Covid-19, in 5 days there would be 3 additional cases for a total of 4 cases, and in another 5 days there would be an additional 9 cases for a total of 13 cases. At the 15-day mark there would be 27 new cases, for a total of 40 cases. 20 days from the first infection there would be 81 new cases, and the progression of new cases every 5 days for 30 days would increase to 243, 729, 2187, 6561, 19,683, to 59,049 new cases at 50 days after the very first infection and a total of 88,573 Covid-19 cases, using this model from WHO. That is, if an infected person only infects 3 other people.

The reproductive number is the average the WHO uses, but we know from current events that the actual number of people an infected person could spread the virus to is often much higher. These higher rates of transmission can account for jumps in the number of virus cases, as we know the rates of infection did not follow only a smooth exponential curve sharply upward, but sometimes increased by much more than the expected reproductive number. The reason for these jumps in transmission of the virus is part of the basis for the reasoning that social distancing is important and wearing masks is important, not because it eliminates the cause of the spread of the virus but because it keeps the spread of the virus at the expected average number of reproductions per person the WHO is using to estimate the outcomes and needs of the infected population.

Another way of understanding the exponential growth of Covid-19 infections is to multiply the number of cases for the previous day by some constant greater than 1 to come up with the number of current cases. This number is not totally consistent, but there is a range of this constant which is 1.15 to 1.25. This means that at day 50, with the example of estimated infections using the WHO reproductive number of 3, we have 88,573 cases, and multiplying this number by the constant in the range of 1.15 to 1.25 we can predict the number of cases on day 51 to be somewhere between 101,859 and 110,716. We can then take that range and reconcile it with the actual number of cases recorded on that day. The estimates can be used as both a model of expected outcomes and to see if the constant is going up or down. In other words, we find out when the number of cases is higher than the range or lower than the range, if efforts to decrease the spread of the virus are helping. In the exponential growth of viral infections, what causes the number of new cases of the virus are the existing cases of the virus. No matter how high or low the range of the constant or the reproductive number, more existing cases will result in more new cases. It sounds repetitive but it is worth examining. If Nd is the number of cases on a given day, and E is the number of people each infected person is exposed to on a given day, and P is the probability of the exposed people becoming infected with the virus, then ΔNd+1= EPNd. As N gets bigger, the function of N gets bigger. The variable of N can actually be factored out of the equation to reveal a simplified equation of EP multiplied by some constant greater than 1, as with the constant which has been discovered to be at 1.15-1.25 with Covid-19.

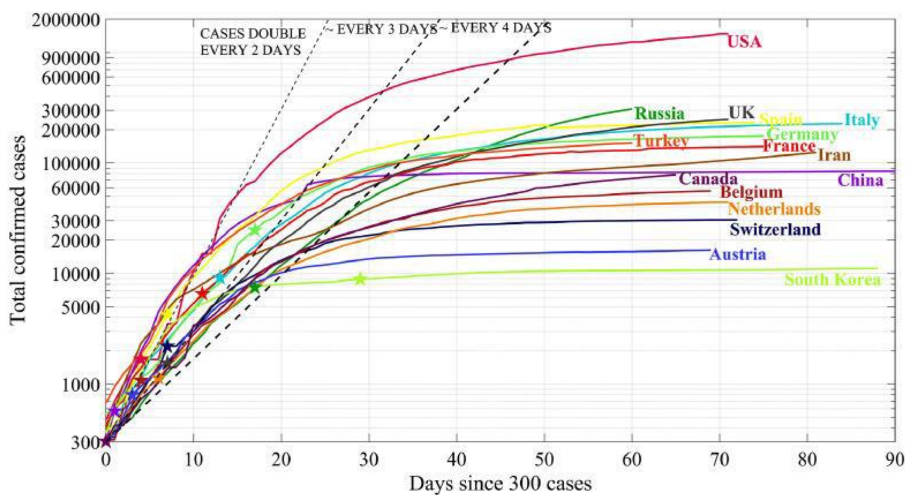
However, what is unique about this understanding of the equation is that it draws attention to the importance of the variables E and P. E is the number of people the infected person exposed to the virus and P is the probability of the exposed person being infected. With Covid-19 we know the rates of infection are very high with exposure. The probability is high, because the virus is very contagious. P is hard to control.

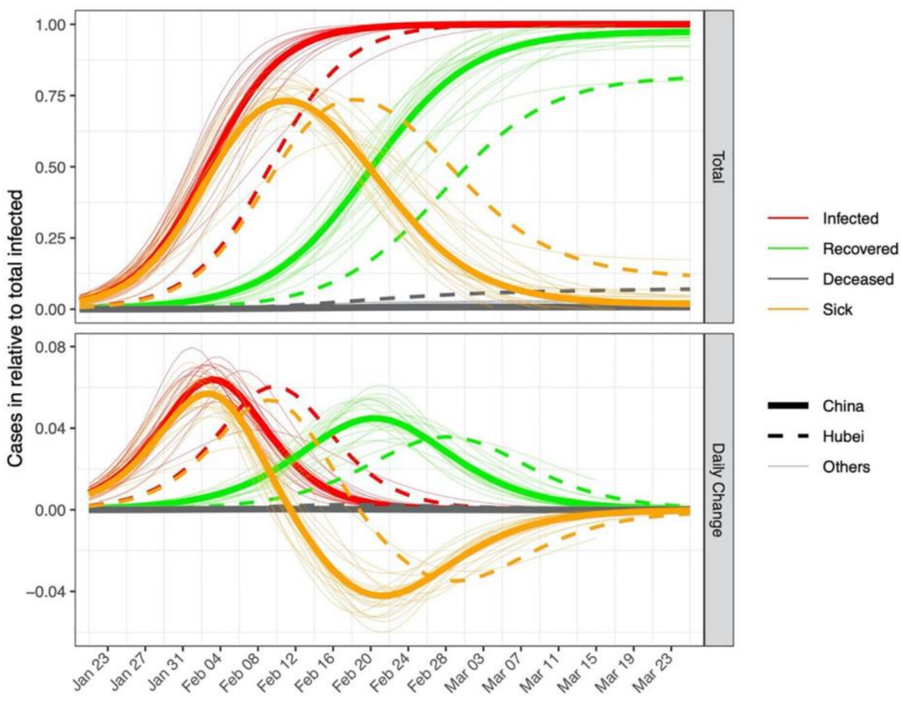
In contrast, E is one variable over which we do have a lot of control. Social distancing and isolation dramatically reduce E. By reducing E we can reduce the resultant infections, and the overage range of the constant. This is the math of flattening the curve. Because an exponential function is the inverse of a logarithmic function, we can use a logarithmic equation to view Covid-19 spread in a different way. The line of linear regression on a Log10 graph of a Covid-19 exponential equation can show us the increase in Covid-19 cases every 10 days. We can much more easily determine this number on a Log10 graph than on an exponential curve, because the number is just the slope of the linear regression line between two points on the line 10 days apart. Another interesting feature of the exponential function in the context of viruses or anything else which affects the population of the world is that at some point, the exponential function stops increasing exponentially, and reproductive numbers and a constant greater than 1 can no longer be used to accurately depict the spread. The reason for this is that the population of the world is limited whereas an exponential function continues into infinity positive.

When the exponential function starts to near the population of the earth, the predictions of the spread of the virus are only realistic if the mathematician uses a logistic curve to reflect the inability of the exponential function to exceed the earth's population. Another way of thinking about this is if the curve flattened naturally because there were no people left on earth who had not already been infected with the virus. In viruses where having been infected once creates almost certain immunity, this logistic curve can be used to predict the natural flattening of the curve in smaller populations, such as within a quarantined city or town or within the borders of a country banned form global travel. The logistic curve is useful in outcomes involving Herd Immunity, but we do not know if it applies broadly to Covid-19 or if the data varies between individuals and strands of the virus. Herd Immunity therefore is widely considered in the mathematic and scientific communities as a false sense of security and a poor reason to eschew making every effort to control the variable of E.

Through this project, I have learned that measures we take to protect ourselves and others from exposure to the Covid-19 virus is of literally exponential importance. The impact of variable E is the only card we have to play until the administration of a vaccine. 39% of the people who contract Covid-19 are at risk for losing their life to the virus. Using basic math and the reproductive number of 3, every individual who is infected with the virus has a life in their hands not their own. That would be the 39% of the 3 people they are likely to infect. It is excellent motivation to social distance, wear masks in every social interaction, and sanitize to prevent the spread of Covid-19.

Visual Representations





Cases in relative to total infected
1.00
0.75
0.50
0.25
0.00
0.08
0.04
0.00
−0.04
Total
Daily Change
Jan 23
Jan 27
Jan 31
Feb 04
Feb 08
Feb 12
Feb 16
Feb 20
Feb 24
Feb 28
Mar 03
Mar 07
Mar 11
Mar 15
Mar 19
Mar 23
Infected
Recovered
Deceased
Sick
China
Hubei
Others